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Locomotion of snakes: a mechanical 'explanation'

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Abstract

This paper presents a multibody model of a snake, a model which incorporates a particular assumption about the interaction between the snake and the ground. Solutions of the associated equations of motion produce results consistent with the motion of a real snake, thus lending credibility to the claim that the postulated snake–ground interaction force law has a basis in reality. © 2000 Elsevier Science Ltd. All rights reserved.

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1. Introduction

In order to move from place to place, a snake deforms its body in such a way as to cause the ground to exert propulsive forces on the body. What *is* this way? Mosauer (1932) noted that "serpentine wriggling, the method used by most snakes for rapid locomotion ... is produced by the progression of transverse waves over the body of the snake from the head to the tail ..." Gray (1946) in a long, often-cited paper, went beyond verbal a posteriori descriptions of snake motions and gave a qualitative mechanical discussion of the phenomenon. However, although it contains numerous force diagrams and sketches of snake segments in various relative orientations, Gray's paper fails to make a rigorous connection between the laws of mechanics and the motion of snakes.

More recently, Miller (1988), for the purpose of producing computer graphics animations of snakes and worms, applied Newton's second law to a mechanical system intended to represent a snake or worm. The system consisted of a complex array of spring-connected particles; the natural lengths of the springs were specified functions of time; and the snake–ground (or worm-ground) interaction was represented by a 'directional' friction force acting on each particle in such a way as to prevent backward

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sliding. Unfortunately, the system under consideration was not described in detail, and its equations of motion were not reported. But his approach enabled Miller to arrive at realistic-looking snake animations on the computer screen.

In the present work, we address the question of how snakes move by: (1) creating a multibody system to represent a snake, (2) postulating relative motions of the bodies of this system, (3) postulating a ground–snake interaction force law, (4) appealing to a principle of mechanics to formulate differential equations of motion, (5) solving these equations numerically, and (6) creating animations and plots based on numerical integration results and assessing the extent to which such animations and plots are consonant with observed motions of snakes.

What is the purpose of this work? It is to create a mathematical description of locomotion of snakes that is in concord with laws of mechanics. Is this useful? We do not claim that it is. But we contend that it explains locomotion of snakes in the sense that it reveals one way in which a snake can move the parts of its body relative to each other so as to accomplish locomotion, that is, motion of the snake from one place to another, and that it lends credibility to the contention that our postulated snake–ground interaction force law is realistic.

Sections 2–7 of what follows deal with items (1)–(6), above. Section 8 contains some concluding remarks.

Throughout the sequel, we follow certain notational conventions of the (commercially available) symbol manipulation computer program AUTOLEV, which program we used to generate differential equations of motion and to create a computer program for the numerical solution of these equations. For example, to distinguish scalars from vectors, we append the symbol ' > ' to characters naming a vector, so that, for instance, A1 > denotes a vector often written $\mathbf{A}_1$ in technical literature. However, since all symbols are defined upon their first appearance, the reader need not have any familiarity with AUTOLEV. Most importantly, we have employed AUTOLEV notational conventions in the text of the paper in lieu of more conventional ones because this makes it unnecessary for an analyst to suffer the inconvenience of having to use two sets of symbols in a given analysis — one set for pencil and paper work and a second set for writing computer programs incorporating equations generated in the hand analysis. In addition, AUTOLEV character strings often are mnemonically superior to common mathematical symbols in describing physical quantities. For example, it is advantageous to denote the angle between lines A and B by AB, rather than by the more commonly employed symbol θ. Furthermore, AUTOLEV makes it possible to use symbols that are more than a single character in length. For example, the aforementioned symbol AB could not be used as the name of an angle in standard mathematical notation because the juxtaposition of A and B necessarily implies the operation of multiplication, that is, A times B, whether or not one desires it to do this. In contrast, AUTOLEV makes a distinction between the single 'name' AB and the product A*B of two names A and B.

2. Multibody model of a snake

In Fig. 1, A, B, …, G designate identical, uniform, hinge-connected rods of mass M and length L. To characterize the configuration of this system in a Newtonian reference frame N in which the axes X1 and X2 of a rectangular Cartesian coordinate system with origin O are fixed, we use the coordinates HE1 and HE2 of point HE (which represents the head of a snake) and the angles NA, AB, BC, …, FG.

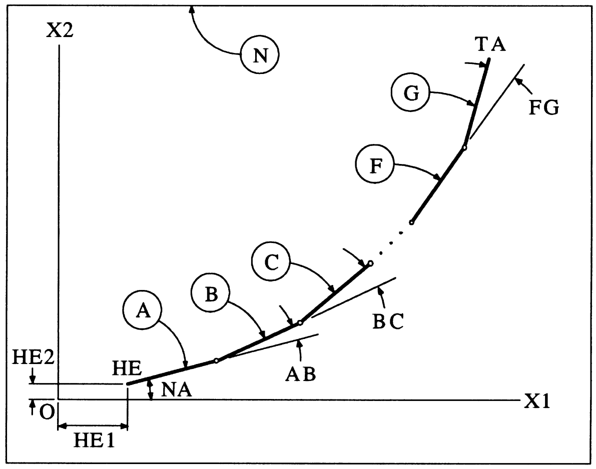

Fig. 1. Seven-body model of a snake.

3. Relative motions of A, ..., G

The orientation of B in A depends solely on the angle AB (see Fig. 1), that of C in B on the angle BC, and so forth. Denoting two constants having the dimensions of length by AMP and LAM, a constant with the dimension of length per unit of time by S, the pure number π by PI, and time by T, we express AB, BC, ..., FG as functions of T and the parameters AMP, LAM, and S as follows:

$$AB = AMP * COS(2 * PI/LAM * (1 * L - S * T))$$
$$BC = AMP * (COS(2 * PI/LAM * (2 * L - S * T)) - COS(2 * PI/LAM * (1 * L - S * T)))$$
$$\cdots\cdots\cdots$$
$$FG = AMP * (COS(2 * PI/LAM * (6 * L - S * T)) - COS(2 * PI/LAM * (5 * L - S * T)))$$

AMP, LAM, and S thus play the roles of the amplitude, the wavelength, and the propagation speed of a harmonic wave.

4. Ground/snake interaction force law

To provide for the interaction of the ground with the snake, we regard the ground as a horizontal plane that exerts on, say, body A a set of forces equivalent to a single force applied to A at AO, the midpoint of A, and given by

$$M * G * \left[- V\_AO\_N > \cdot (MU1 * A1 > *A1 > +MU2 * A2 > *A2 >) + A3 > \right]$$

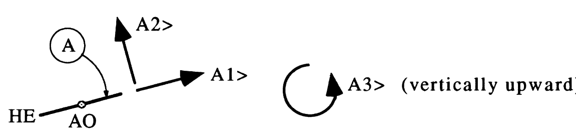

Fig. 2. Unit vectors fixed in body A.

where G is the local acceleration of gravity, V_AO_N> is the velocity of AO in N, MU1 and MU2 are constants, and A1 > , A2 > , A3> are mutually perpendicular unit vectors fixed in A and directed as in Fig. 2. Similarly, we let the ground exert on B a force

$$\mathrm{M} * \mathrm{G} * \left[-\mathrm{V\_BO\_N} > \cdot (\mathrm{MU1} * \mathrm{B1} > *\mathrm{B1} > +\mathrm{MU2} * \mathrm{B2} > *\mathrm{B2} >) + \mathrm{B3} > \right]$$

and so forth for C, ..., G. This is, in essence, a 'viscous' friction force law with different damping constants in the directions parallel and perpendicular to each rod. Whether or not this characterizes the actual interaction between a snake and the ground can be determined only by comparing results of a motion simulation based on this postulate with the actual motion of a snake. We shall return to this point in Section 7.

5. Differential equations of motion

The instantaneous motion of the system can be characterized by generalized speeds U1, U2, U3 defined as the first time-derivatives of HE1, HE2, and NA, respectively. Hence, we have the kinematical differential equations

$$\frac{\mathrm{dHE1}}{\mathrm{dT}} = \mathrm{U1}, \quad \frac{\mathrm{dHE2}}{\mathrm{dT}} = \mathrm{U2}, \quad \frac{\mathrm{dNA}}{\mathrm{dT}} = \mathrm{U3}$$

To formulate the associated dynamical differential equations, we appeal to the motion law

$$\mathrm{F_r} + \mathrm{F_r^*} = 0 \quad (\mathrm{r} = 1, 2, 3)$$

where $\mathrm{F_r}$ and $\mathrm{F_r^*}$ are the generalized active force and the generalized inertia force corresponding to Ur (r = 1, 2, 3). This leads us with the aid of AUTOLEV (the AUTOLEV commands file appears in Appendix A) to the equations

$$\frac{\mathrm{dU1}}{\mathrm{dT}} = \mathrm{Z320}, \quad \frac{\mathrm{dU2}}{\mathrm{dT}} = -\mathrm{Z321}, \quad \frac{\mathrm{dU3}}{\mathrm{dT}} = -\mathrm{Z322}$$

where Z320, Z321, and Z322 are complicated functions of U1, U2, U3, and T. To be specific, Z320, for example, is defined as (Z315*Z312 + Z317*Z310−Z318*Z311)/Z316, where Z315 is given by 0.5*Z300*Z304 + 0.5*Z301*Z303, and so on; and U1, U2, U3 come into evidence explicitly, for example, in the definition of Z127 as Z126*(L*U3 + 2*Z1*U2−2*Z2*U1), where Z126, Z1, and Z2 stand for G*M*MU2, COS(NA), and SIN(NA), respectively. As should now be evident, it is the introduction of the Zs (accomplished automatically by AUTOLEV) that makes it possible to express the dynamical differential equations in a manageable form.

6. Numerical solution of the equations of motion

When the AUTOLEV commands file listed in Appendix A is executed, the last line causes AUTOLEV to write two files. One of these is a FORTRAN program, called snake.for, that enables one to perform simultaneously numerical integrations of the kinematical and the dynamical differential equations of motion; the other is an input file for snake.for, called snake.in. A listing of snake.in appears in Fig. 3.

```
File: snake.in
------------------------------------------------------------------------
Replace this line with a message to appear on all output files.
------------------------------------------------------------+-----------
Description          Quantity             Units             | Value
------------------------------------------------------------|-----------
Constant:            AMP                  DEG               |  30
Constant:            C                    M/S               |  2.0
Constant:            G                    M/S^2             |  9.8
Constant:            L                    M                 |  0.5
Constant:            LAM                  M                 |  2.4
Constant:            M                    KG                |  0.1
Constant:            MU1                  S/M               |  0.01
Constant:            MU2                  S/M               |  0.20

Initial Value:       NA                   DEG               |  0.0
Initial Value:       U1                   M/S               |  0.0
Initial Value:       U2                   M/S               |  0.0
Initial Value:       U3                   RAD/S             |  0.0
Initial Value:       HE1                  M                 |  0.0
Initial Value:       HE2                  M                 |  0.0

Initial Time:        TINITIAL             S                 |  0.0
Final Time:          TFINAL               S                 |  30.0
Integration Step:    INTEGSTP             S                 |  0.1
Print-Integer:       PRINTINT             Positive Integer  |  1
Absolute Error:      ABSERR                                 |  1.0E-08
Relative Error:      RELERR                                 |  1.0E-07
------------------------------------------------------------+-----------
```

Fig. 3. Input file created by AUTOLEV for AUTOLEV-generated FORTRAN program.

Execution of snake.for brings into existence output files called snake.1, …, snake.8. In snake.1, values of T and associated values of HE1, HE2, TA1, and TA2 are recorded (TA1 and TA2 are the Cartesian coordinates of point TA, which, as can be seen in Fig. 1, represents the tip of the snake's tail), while snake.2, …, snake.8 contain data used to drive animations created with the aid of the (commercially available) program ANIMAKE. Such animations furnish a convenient basis for assessing the realism of the theory under consideration. In print, the best we can do is to show plots based on data recorded in the output file snake.1.

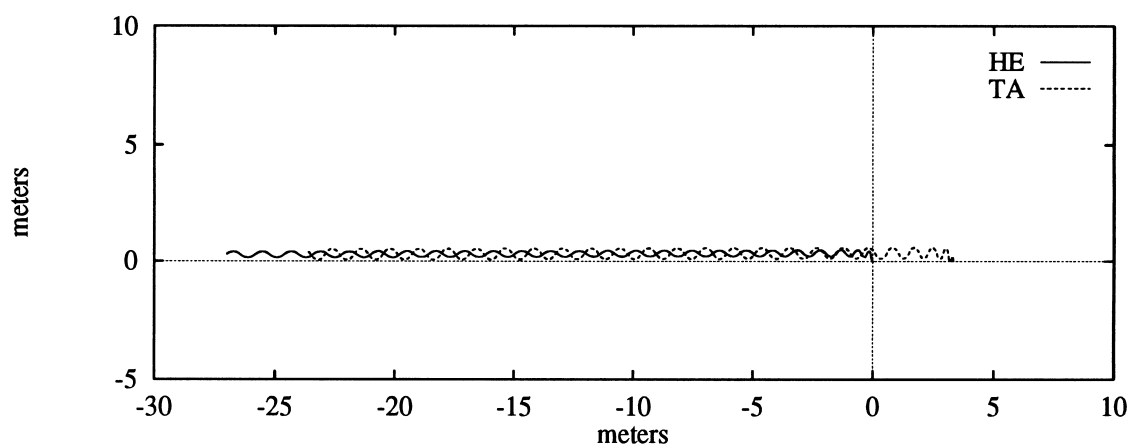

Fig. 4. Paths of HE and TA.

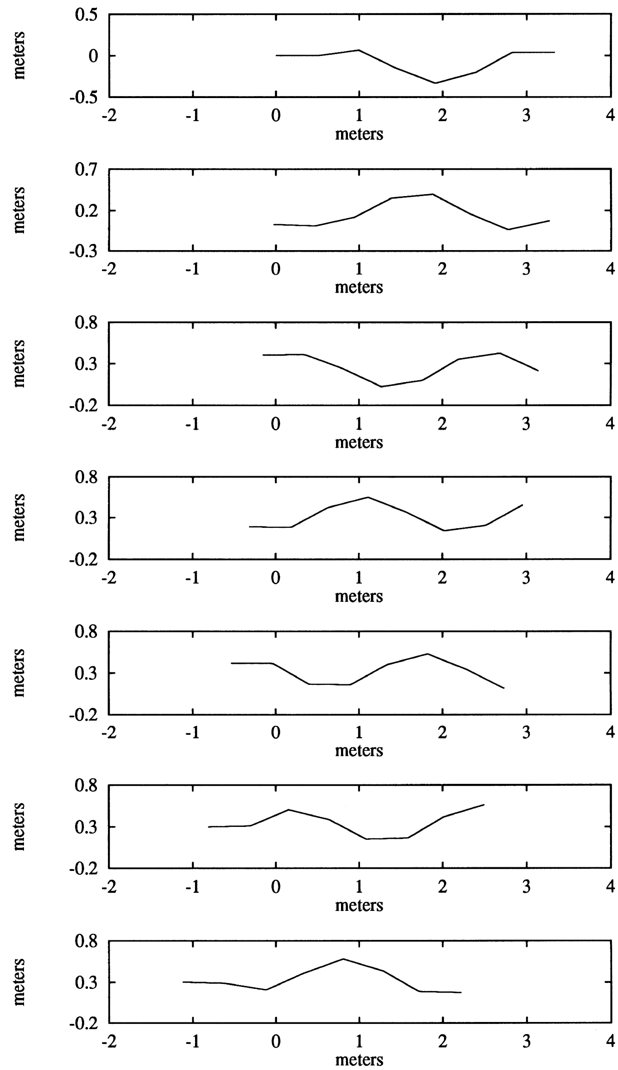

Fig. 5. Representative stills from the animation.

7. Representative results

The curves in Fig. 4 show the paths on which HE and TA move when the system parameters and the initial values of HE1, HE2, NA, U1, U2, and U3 have the values recorded in Fig. 3. Clearly, these paths are consistent with essentially rectilinear locomotion of the system under consideration. An animation based on the same data lends support to the contention that such motions can be brought about by the postulated relative motions of body parts and snake–ground interaction force law. Fig. 5 shows seven representative stills from the animation, these corresponding from top to bottom to the configuration of the system at t = 0.0, 0.5, 1.0, 1.5, 2.0, 2.5, and 3.0 s, respectively.

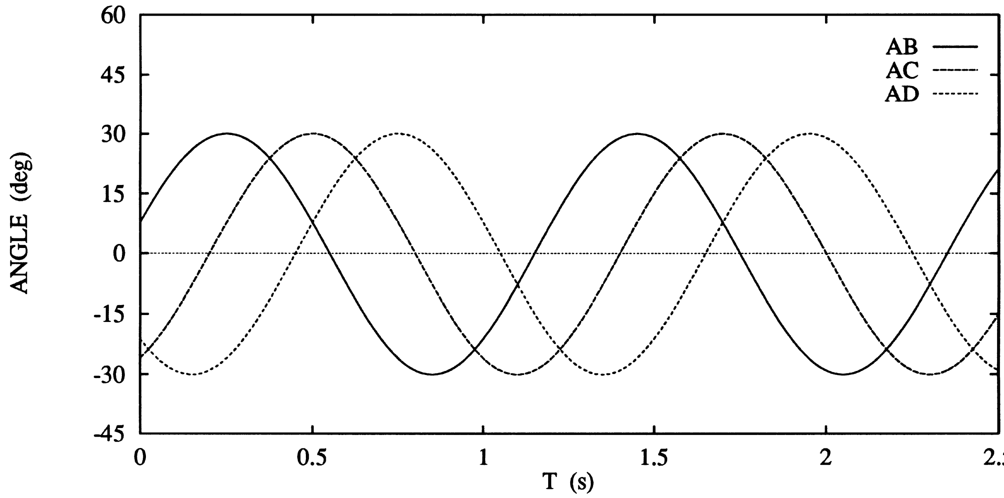

Fig. 6. Angles AB, AC, and AD.

8. Conclusion

What the multibody snake does to propel itself from place to place can be described most simply in terms of the angles between A and B, A and C, and so forth. The first three of these, which we call AB, AC, and AD, are plotted as a function of T in Fig. 6 (plots of AE, AF, and AG are similar), which shows that they are sinusoids differing from each other only as regards to phase; and the phase differences are precisely those associated with a wave traveling from HE toward TA. This, then, together with our postulated snake–ground interaction force law (see Section 4), is our explanation of locomotion of snakes.

Appendix A

```
% SNAKE.AL
AUTOZ ON
NEWTONIAN N
POINTS O
CONSTANTS G,L,S,AMP,LAM,MU1,MU2
BODIES A,B,C,D,E,F,G
MASS A = M,B = M,C = M,D = M,E = M,F = M,G = M
J = M*L^2/12
INERTIA A,0,0,J
INERTIA B,0,0,J
INERTIA C,0,0,J
INERTIA D,0,0,J
INERTIA E,0,0,J
INERTIA F,0,0,J
INERTIA G,0,0,J
VARIABLES HE{2}',NA',U{3}'
```

```
HE1'=U1
HE2'=U2
NA'=U3
AB=AMP*COS(2*PI/LAM*(1*L-S*T))
BC=AMP*(COS(2*PI/LAM*(2*L-S*T))-COS(2*PI/LAM*(1*L-S*T)))
CD=AMP*(COS(2*PI/LAM*(3*L-S*T))-COS(2*PI/LAM*(2*L-S*T)))
DE=AMP*(COS(2*PI/LAM*(4*L-S*T))-COS(2*PI/LAM*(3*L-S*T)))
EF=AMP*(COS(2*PI/LAM*(5*L-S*T))−COS(2*PI/LAM*(4*L-S*T)))
FG=AMP*(COS(2*PI/LAM*(6*L-S*T))−COS(2*PI/LAM*(5*L-S*T)))
AC=AB+BC
AD=AC+CD
AE=AD+DE
AF=AE+EF
AG=AF+FG
SIMPROT(N,A,3,NA)
SIMPROT(A,B,3,AB)
SIMPROT(A,C,3,AC)
SIMPROT(A,D,3,AD)
SIMPROT(A,E,3,AE)
SIMPROT(A,F,3,AF)
SIMPROT(A,G,3,AG)
POINTS HE,AB,BC,CD,DE,EF,FG,TA
P_O_HE > =HE1*N1 > +HE2*N2 >
P_HE_AO > =0.5*L*A1 >
P_AO_AB > =0.5*L*A1 >
P_AB_BO > =0.5*L*B1 >
P_BO_BC > =0.5*L*B1 >
P_BC_CO > =0.5*L*C1 >
P_CO_CD > =0.5*L*C1 >
P_CD_DO > =0.5*L*D1 >
P_DO_DE > =0.5*L*D1 >
P_DE_EO > =0.5*L*E1 >
P_EO_EF > =0.5*L*E1 >
P_EF_FO > =0.5*L*F1 >
P_FO_FG > =0.5*L*F1 >
P_FG_GO > =0.5*L*G1 >
P_GO_TA > =0.5*L*G1 >
W_A_N > =U3*A3 >
W_B_N > =W_A_N > +DT(AB)*N3 >
W_C_N > =W_A_N > +DT(AC)*N3 >
W_D_N > =W_A_N > +DT(AD)*N3 >
W_E_N > =W_A_N > +DT(AE)*N3 >
W_F_N > =W_A_N > +DT(AF)*N3 >
W_G_N > =W_A_N > +DT(AG)*N3 >
V_HE_N > =U1*N1 > +U2*N2 >
V_AO_N > =V_HE_N > +DT(P_HE_AO > ,N)
V_BO_N > =V_HE_N > +DT(P_HE_BO > ,N)
V_CO_N > =V_HE_N > +DT(P_HE_CO > ,N)
```

```
V_DO_N> = V_HE_N> + DT(P_HE_DO>,N)
V_EO_N> = V_HE_N> + DT(P_HE_EO>,N)
V_FO_N> = V_HE_N> + DT(P_HE_FO>,N)
V_GO_N> = V_HE_N> + DT(P_HE_GO>,N)
EXPRESS(V_AO_N>,A)
EXPRESS(V_BO_N>,B)
EXPRESS(V_CO_N>,C)
EXPRESS(V_DO_N>,D)
EXPRESS(V_EO_N>,E)
EXPRESS(V_FO_N>,F)
EXPRESS(V_GO_N>,G)
FORCE(AO,M*G*DOT(-V_AO_N>,MU1*A1>*A1> + MU2*A2>*A2>))
FORCE(BO,M*G*DOT(-V_BO_N>,MU1*B1>*B1> + MU2*B2>*B2>))
FORCE(CO,M*G*DOT(-V_CO_N>,MU1*C1>*C1> + MU2*C2>*C2>))
FORCE(DO,M*G*DOT(-V_DO_N>,MU1*D1>*D1> + MU2*D2>*D2>))
FORCE(EO,M*G*DOT(-V_EO_N>,MU1*E1>*E1> + MU2*E2>*E2>))
FORCE(FO,M*G*DOT(-V_FO_N>,MU1*F1>*F1> + MU2*F2>*F2>))
FORCE(GO,M*G*DOT(-V_GO_N>,MU1*G1>*G1> + MU2*G2>*G2>))
ZERO = FR() + FRSTAR()
SOLVE(ZERO,U1',U2',U3')
TA1 = DOT(P_O_TA>,N1>)
TA2 = DOT(P_O_TA>,N2>)
UNITS G = M/S^2,[L,LAM,HE1,HE2,TA1,TA2] = M,M = KG,[MU1,MU2] = S/M
UNITS [U1,U2] = M/S,U3 = RAD/S,S = M/S,T = S
UNITS [NA,AB,BC,CD,DE,EF,FG,AMP,AC,AD] = DEG
INPUT AMP = 30,S = 2,G = 9.8,L = 0.5,LAM = 2.4,M = 0.1,MU1 = 0.01,MU2 = 0.2,TFINAL = 30
OUTPUT T,HE1,HE2,TA1,TA2
CODE ODE() SNAKE.FOR,SUBS
```

References

Gray, J., 1946. The mechanism of locomotion in snakes. Journal of Experimental Biology 23 (2), 101–120.
Miller, G., 1988. The motion dynamics of snakes and worms. Computer Graphics 22 (4), 169–178.
Mosauer, W., 1932. On the locomotion of snakes. Science 76, 583–585.